Oceanic Dynamics under Location Uncertainty: Towards a consistent stochastic modeling

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Oceanic Dynamics under Location Uncertainty

Towards a consistent stochastic modelling

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Objective

- **Why stochastic modeling ?**

  → **Take into account small-scale / unresolved processes**

  → **Uncertainty Quantification, Ensemble Forecasts**

- **Why consistent model ?**

  → **Both scales must respect appreciate physical laws**

  → **Losing consistency may provide wrong statistics**

Outline

Location Uncertainty Principles (Mémin 2014; Resseguier et al. 2017)

- **Stochastic flow :**

$$\mathrm{d}\boldsymbol{X}_t = \boldsymbol{u}(\boldsymbol{X}_t, t)\mathrm{d}t + \boldsymbol{\sigma}(\boldsymbol{X}_t, t)\mathrm{d}\boldsymbol{B}_t$$

large-scale resolved — *small-scale unresolved*

- **Functional process :**

$$\boldsymbol{\sigma}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{B}_t = \int_D \breve{\boldsymbol{\sigma}}(\boldsymbol{x}, \boldsymbol{y}, t)\mathrm{d}\boldsymbol{B}_t(\boldsymbol{y})\mathrm{d}\boldsymbol{y}$$

correlated in space, uncorrelated in time — *symmetric kernel* — *space-time white noise*

- **Variance tensor :**

$$\boldsymbol{a} \triangleq \boldsymbol{\sigma}\boldsymbol{\sigma}^T = \mathbb{E}\left[\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t(\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t)^T\right]/\mathrm{d}t$$

homogeneous / heterogeneous

- **Transport of a random tracer :**

$$(\ \boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = 0\ )$$

$$\mathrm{D}_t\theta \triangleq \mathrm{d}_t\theta + \boldsymbol{u}^\star\cdot\boldsymbol{\nabla}\theta\mathrm{d}t + \boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t\cdot\boldsymbol{\nabla}\theta - \frac{1}{2}\boldsymbol{\nabla}\cdot(\boldsymbol{a}\boldsymbol{\nabla}\theta)\mathrm{d}t = 0$$

corrected drift $\boldsymbol{u} - \frac{1}{2}\boldsymbol{\nabla}\cdot\boldsymbol{a}$ — *multiplicative noise* — *subgrid diffusion*

Balanced Energy $(\ \boldsymbol{\nabla}\cdot\boldsymbol{u}^\star = 0\ )$ $\Rightarrow$ $\dfrac{\mathrm{d}}{\mathrm{d}t}\displaystyle\int_D \frac{1}{2}\theta^2 = 0$

Barotropic Quasi–Geostrophic Model under LU (Resseguier et al. 2017; Li et al. 2019)

- **Evolution of the potential vorticity with source processes :**

$$\mathrm{D}_t q = S_1(\boldsymbol{\nabla u})\mathrm{d}t + S_2(\boldsymbol{\nabla u})\mathrm{d}\boldsymbol{B}_t$$

- **Evolution of the stream function :**

$$q = \Delta\psi - \psi/L_R^2 + f$$

- **Strong incompressible constraints :**

$$\boldsymbol{u} = \boldsymbol{\nabla}^\perp\psi, \qquad \boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t = \boldsymbol{\nabla}^\perp\varphi\mathrm{d}B_t, \qquad \boldsymbol{\nabla}\cdot\boldsymbol{u}^\star = 0$$

→ **Conservation of total energy :**

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\int_D \frac{1}{2}\left[\|\boldsymbol{\nabla}\psi\|^2 + (\psi/L_R^2)^2\right] = 0}$$

$$\left[\ \text{Sketch}: \int_D \psi\boldsymbol{\sigma}\circ\mathrm{d}\boldsymbol{B}_t\cdot\boldsymbol{\nabla}(q-f) = \int_D \psi S_2(\boldsymbol{\nabla u})\circ\mathrm{d}\boldsymbol{B}_t\ \right]$$

- **Transport of a prognostic passive tracer :**

$$\mathrm{D}_t q' = 0$$

→ **Conservation of tracer energy :**

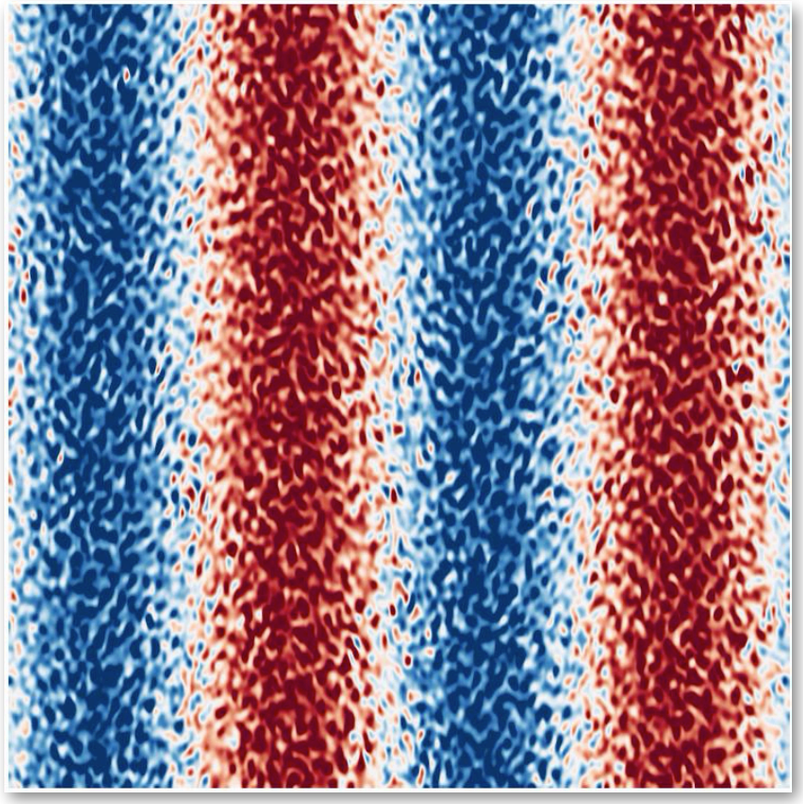
$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\int_D \frac{1}{2}(q')^2 = 0}$$

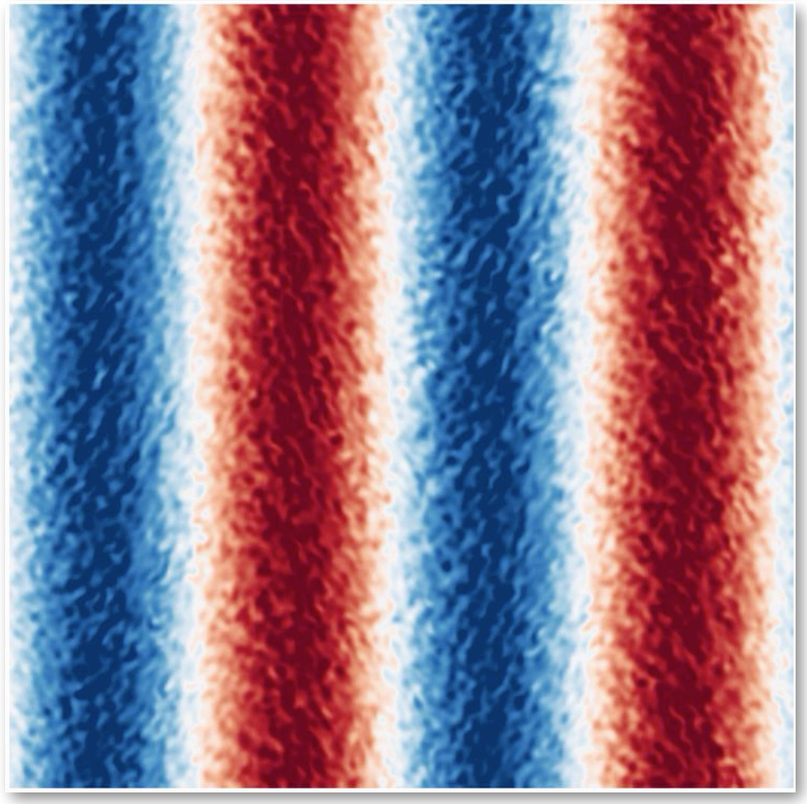
Outline

Structure–preserving of Rossby wave

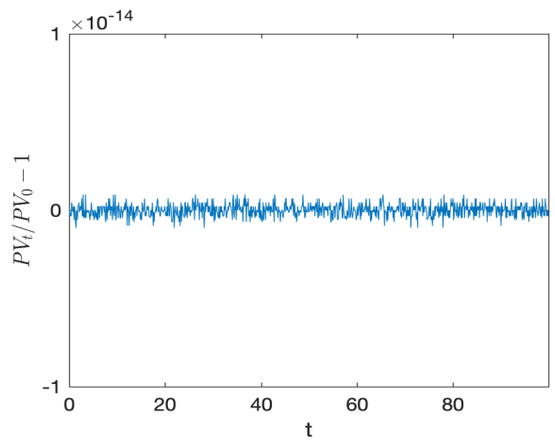
- **Flowchart :** $q \longleftrightarrow \psi \not\longrightarrow q'$

(Potential Vorticity) (Stream Function) (Passive Tracer)

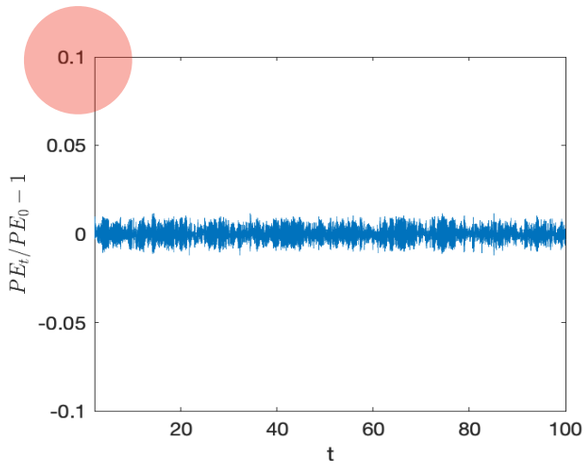




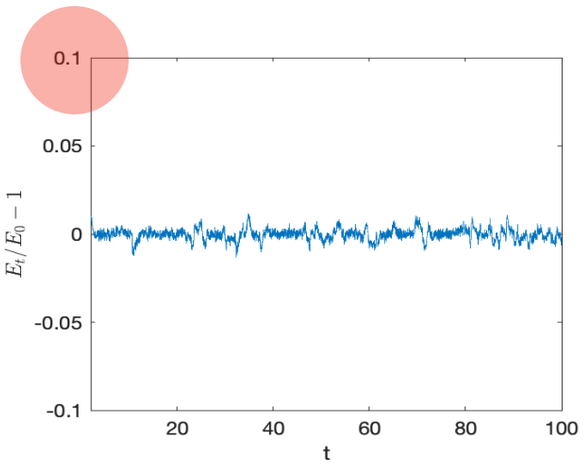
- **Illustration of conservations :**



Passive Tracer



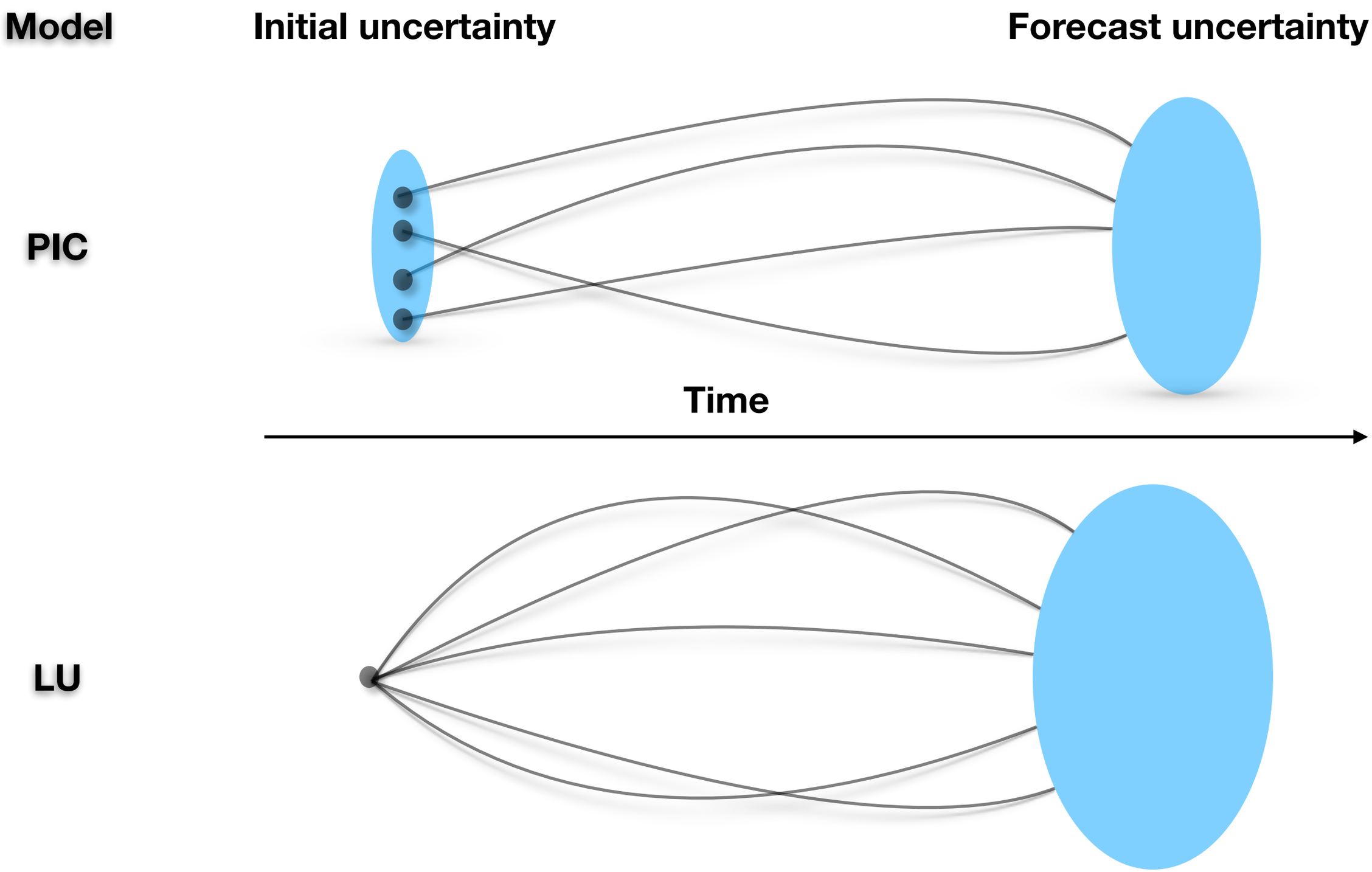
Tracer Energy



Total Energy

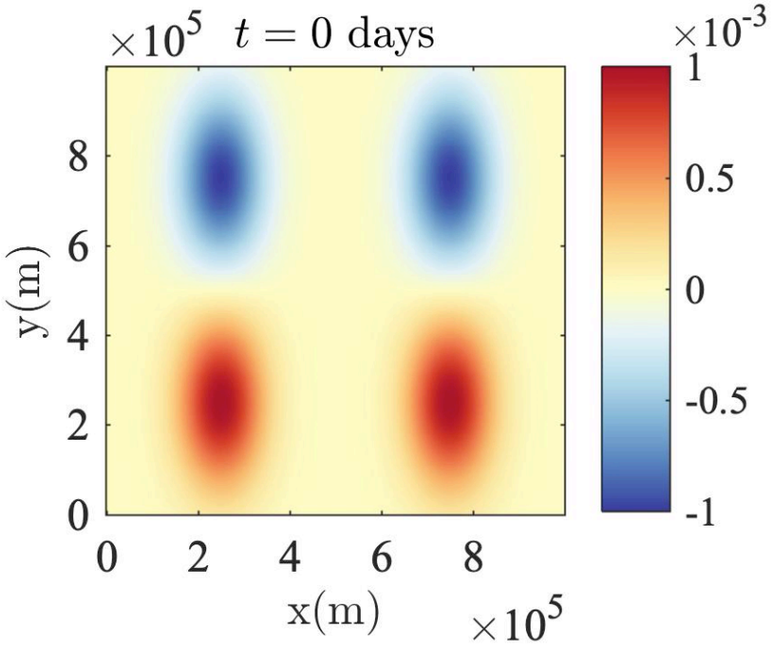
Ensemble forecasting verification of SQG_{MU}

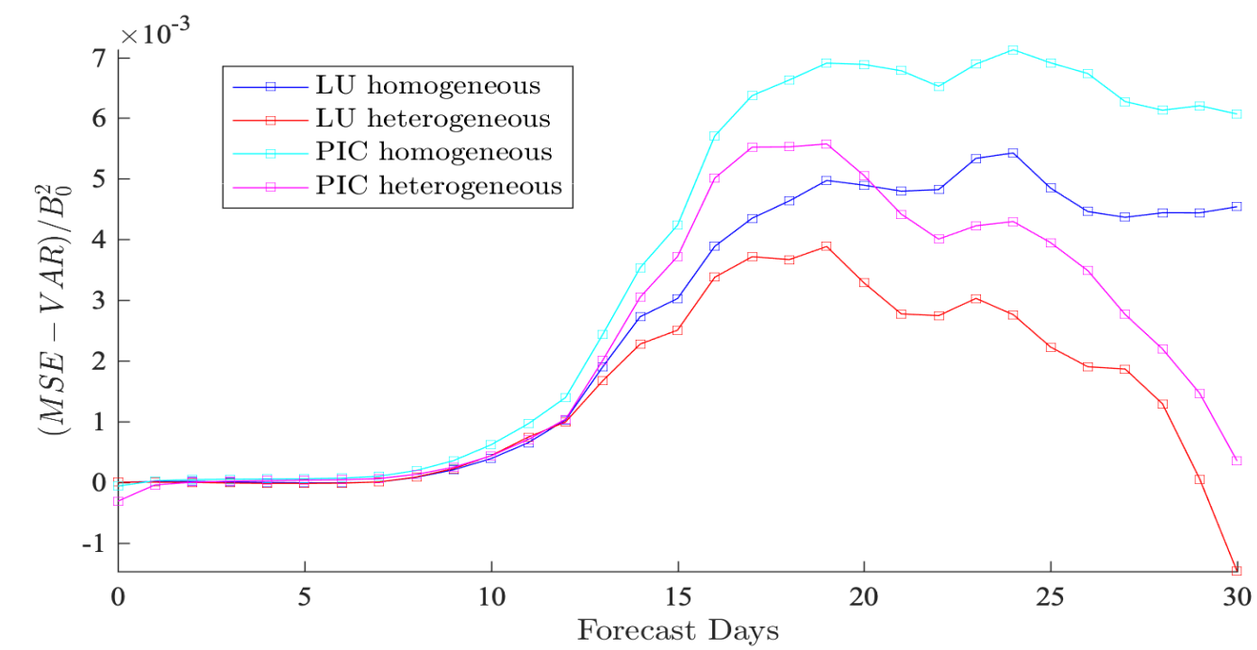
- **Illustration of ensemble prediction systems :**

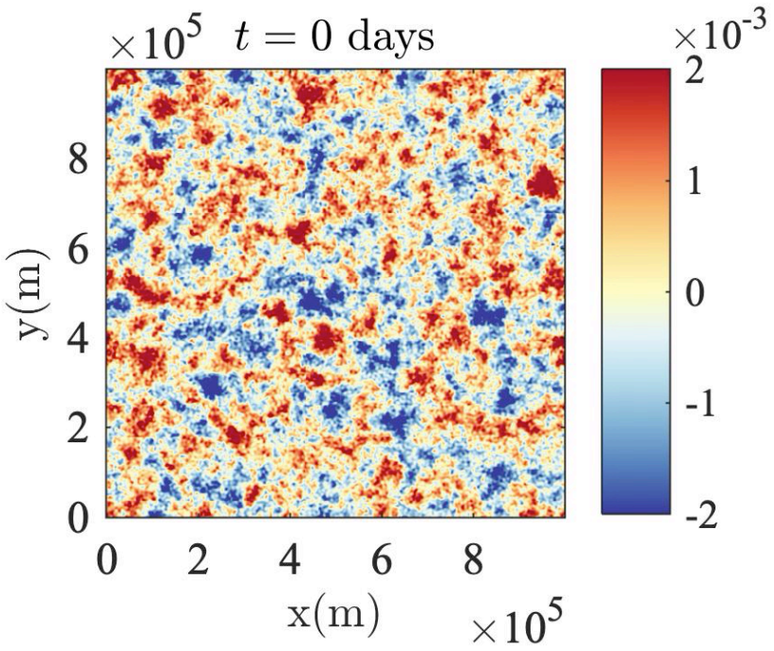


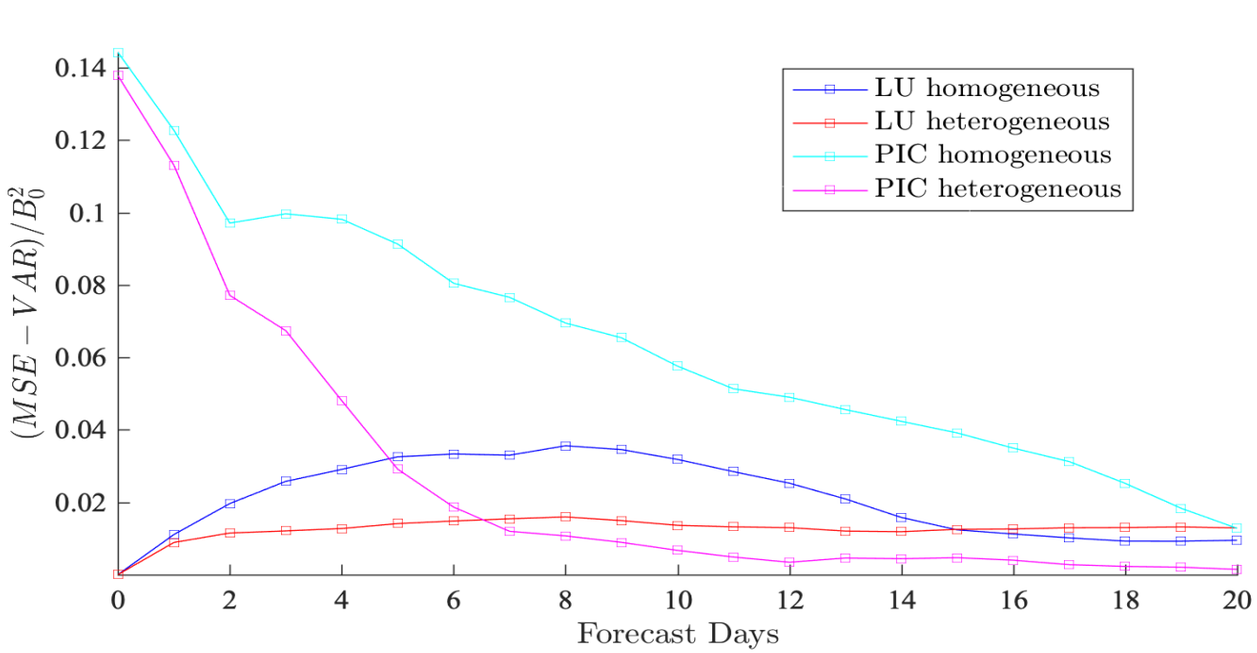
Ensemble forecasting verification of SQG_{MU} (Resseguier et al. 2019)

- **Reliability — Ensemble Spread match MSE of the Ensemble Mean :**



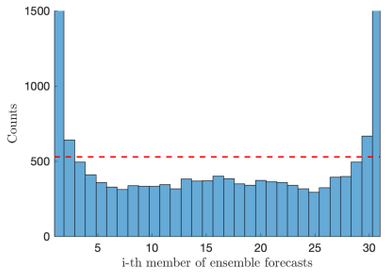
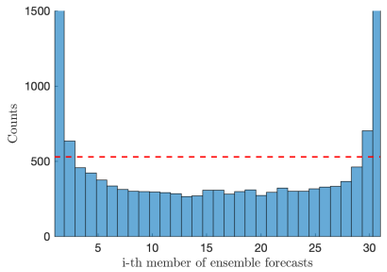
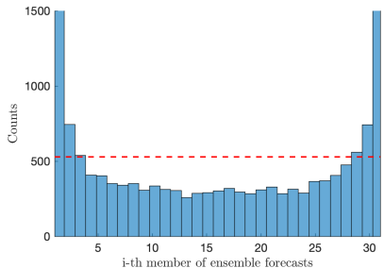
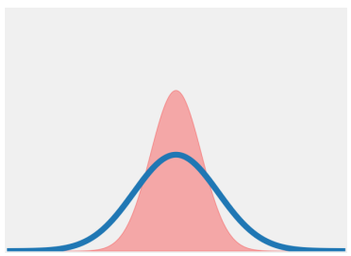
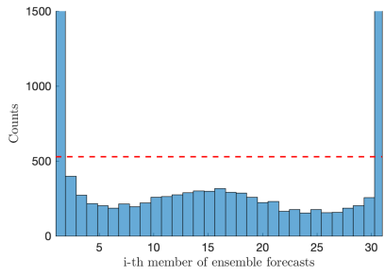
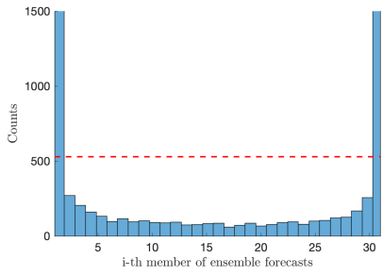
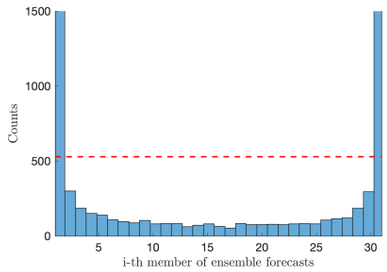
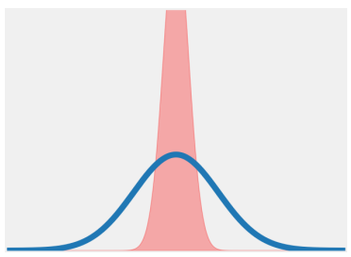
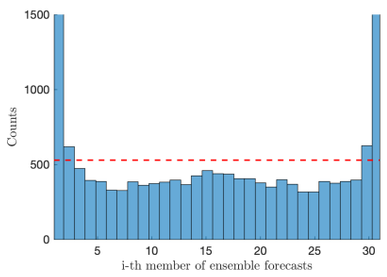
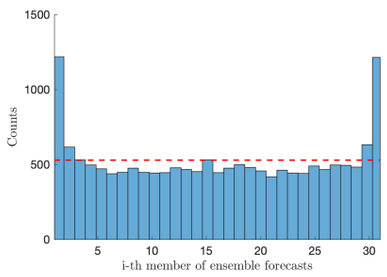
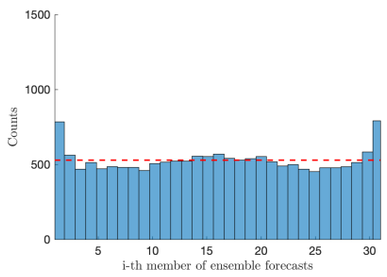
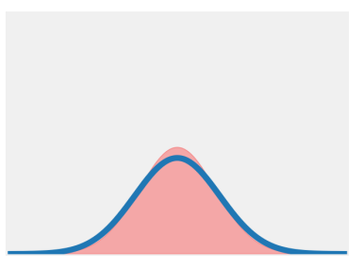
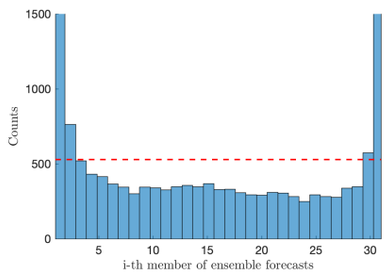
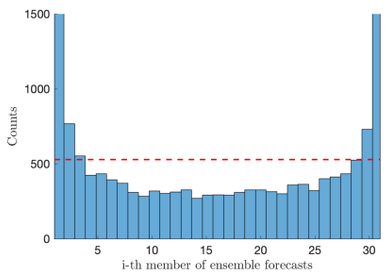
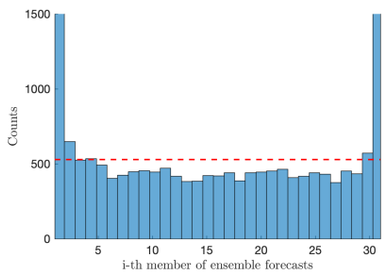
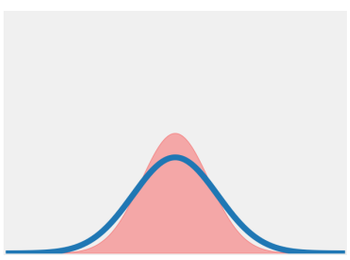




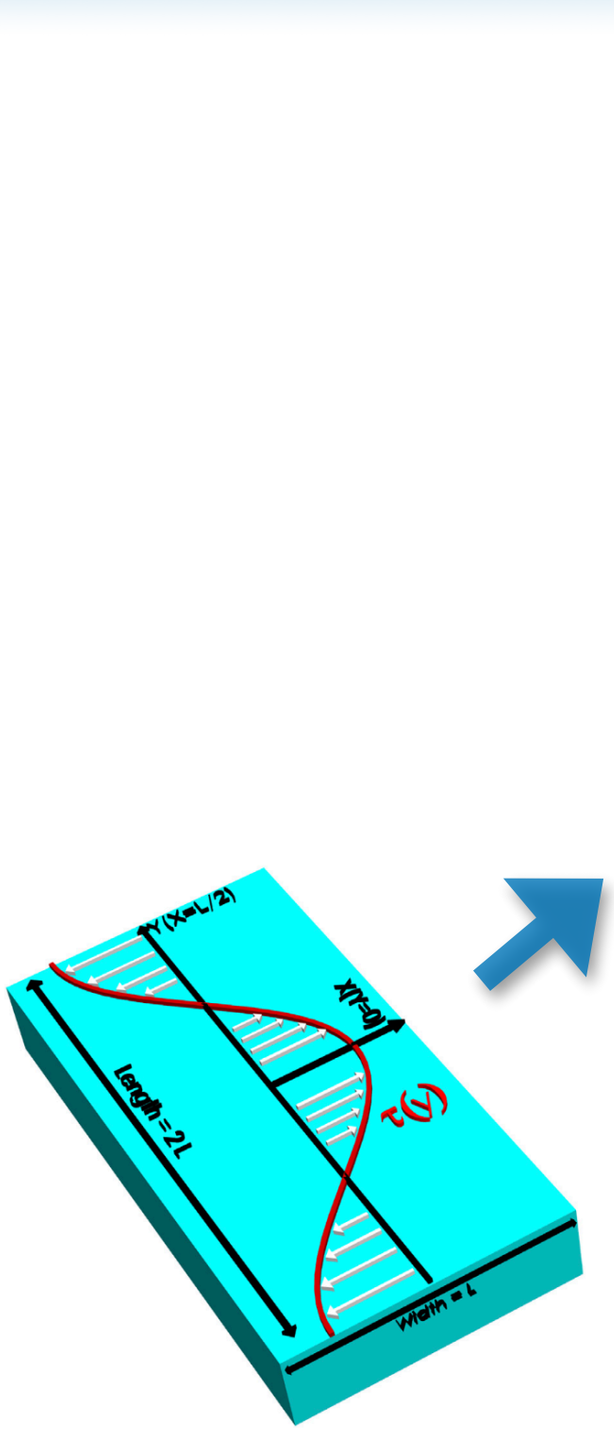


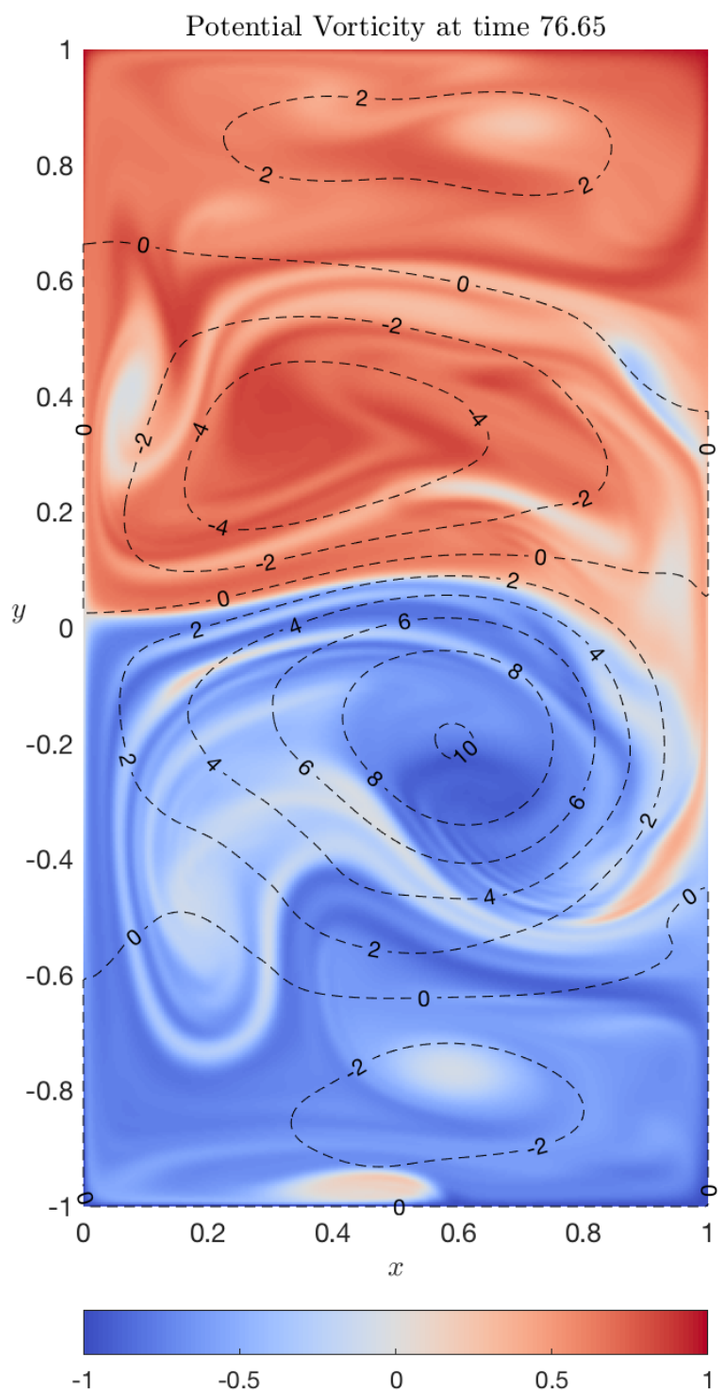
Ensemble forecasting verification of SQG_{MU} (Resseguier et al. 2019)

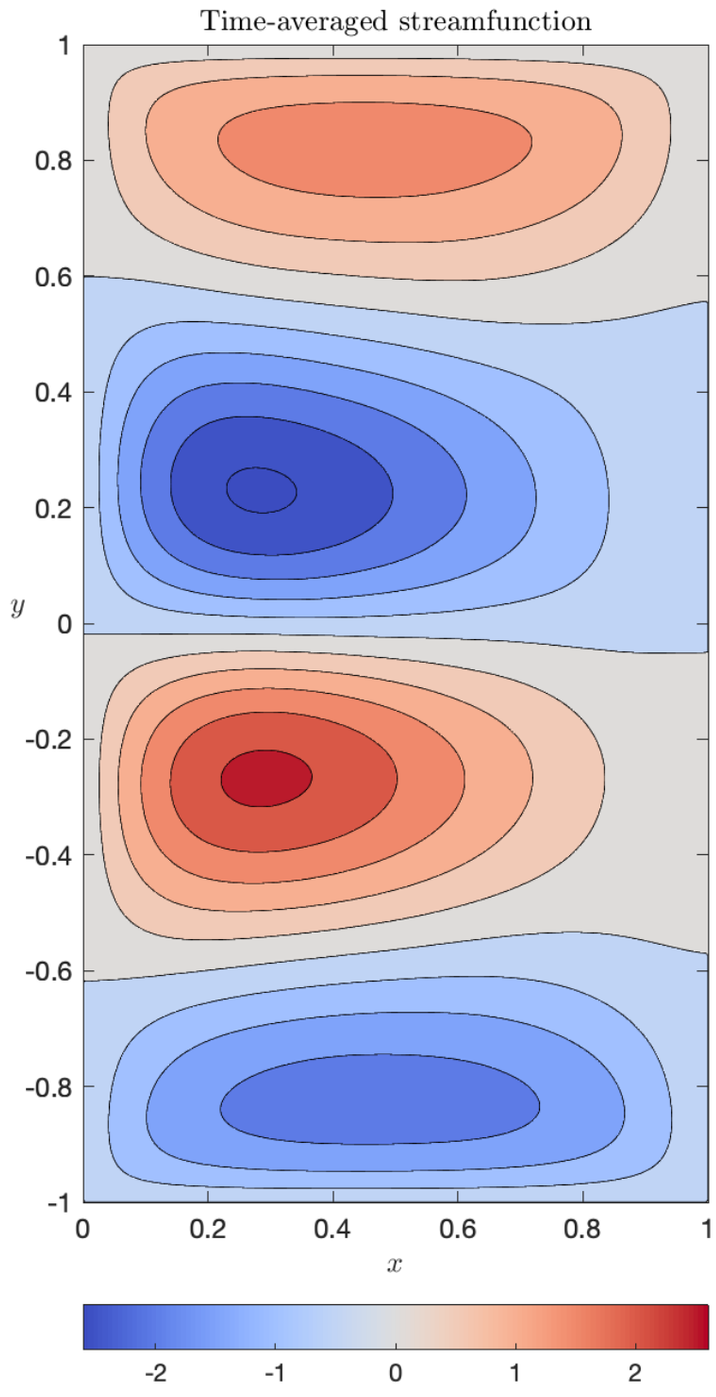
Reliability — Ranked Histogram of Ensemble Members :

Model	Day 10	Day 20	Day 30	Reliability
LU homogeneous				
PIC homogeneous				
LU heterogeneous				
PIC heterogeneous				

Time–statistics of wind–driven circulation

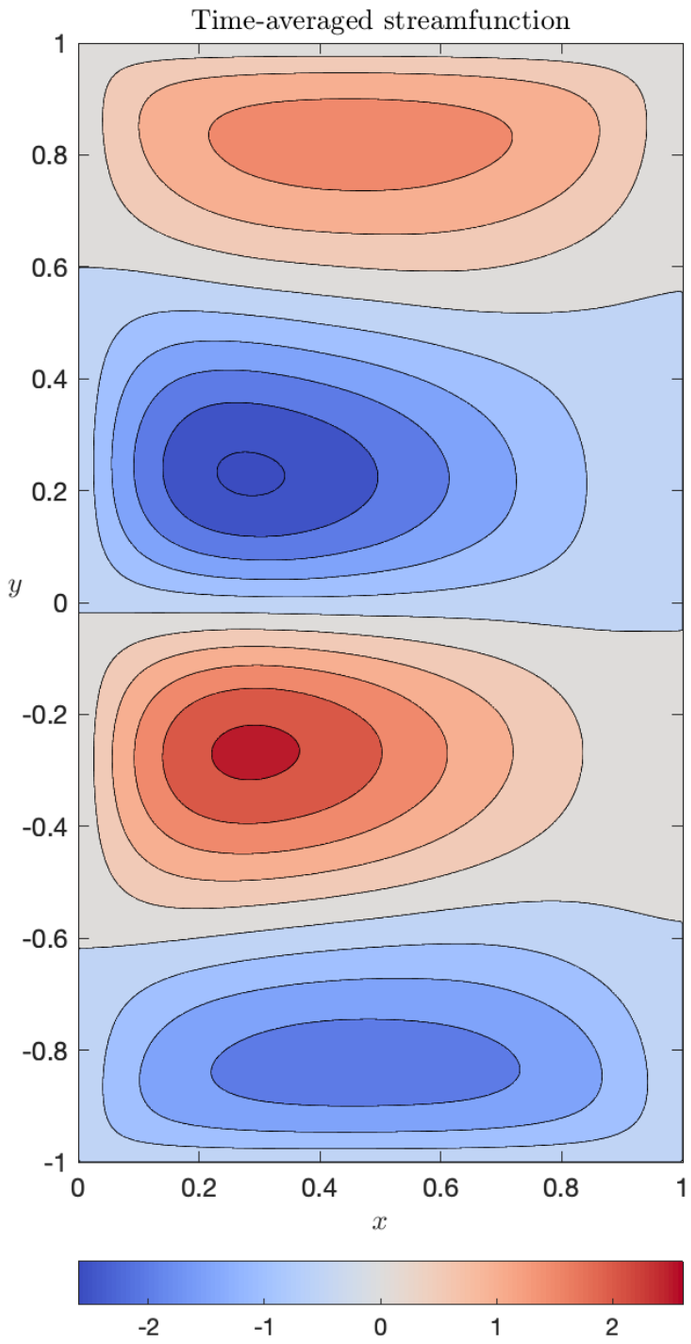




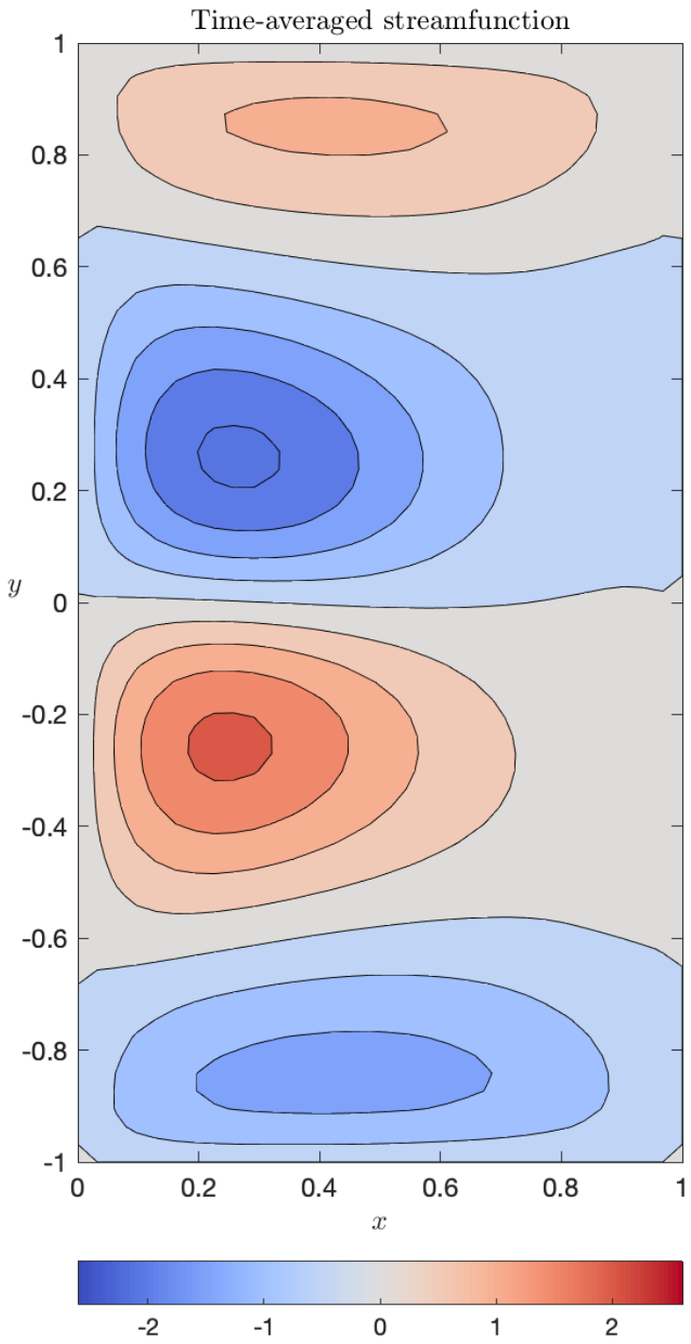


DNS 256×512 with $Re = 450$ and $Ro = 0.0016$

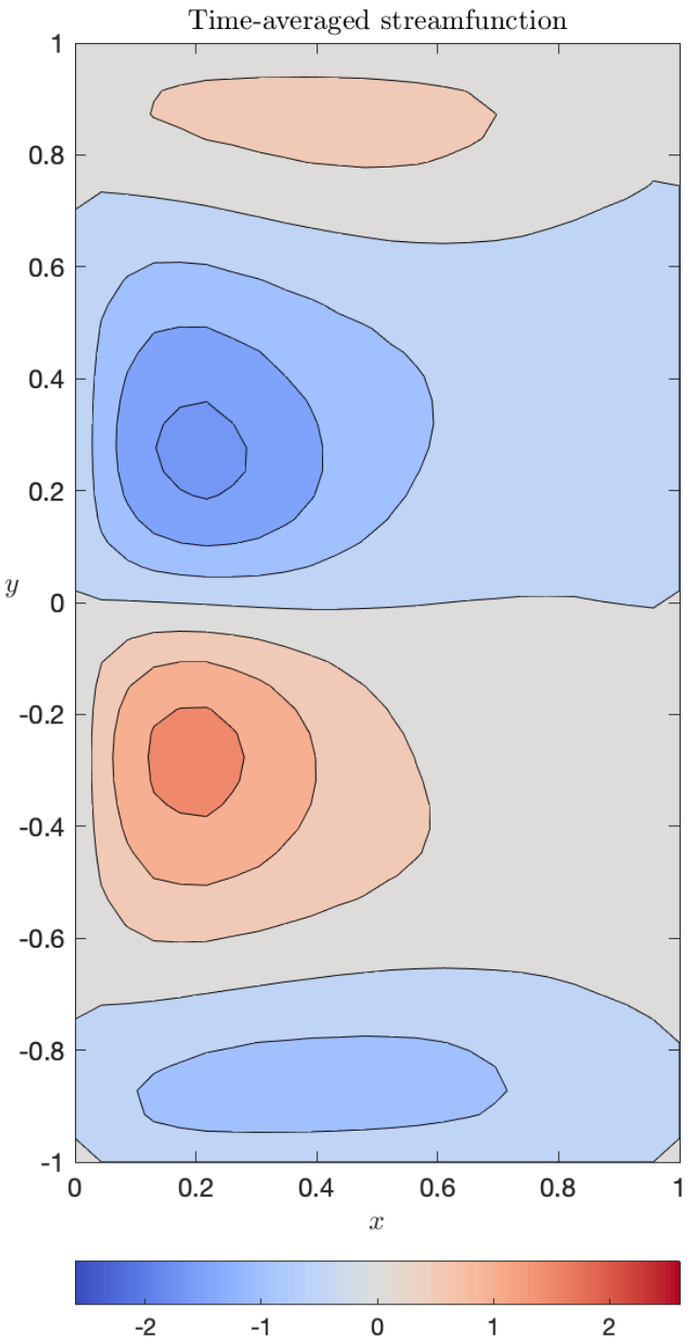
Time–statistics of wind–driven circulation



DNS 256×512



LU 32×64



LES_{LU} 32×64

Truth ←

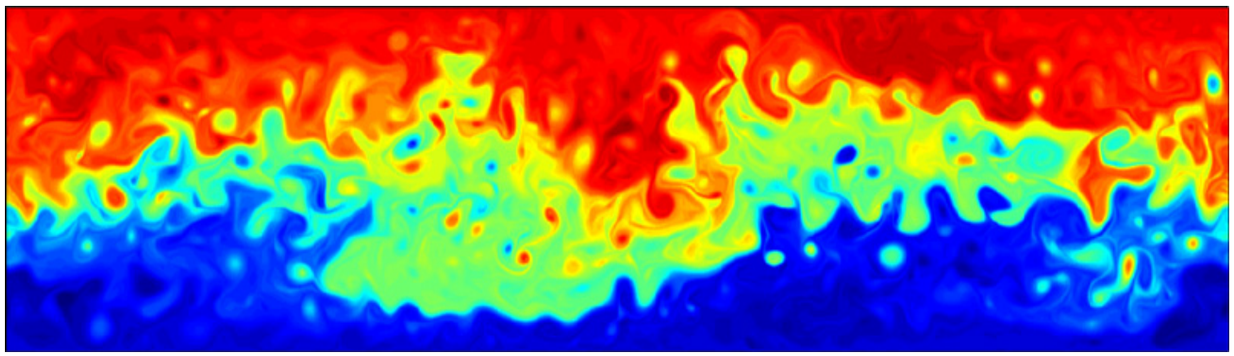
Outline

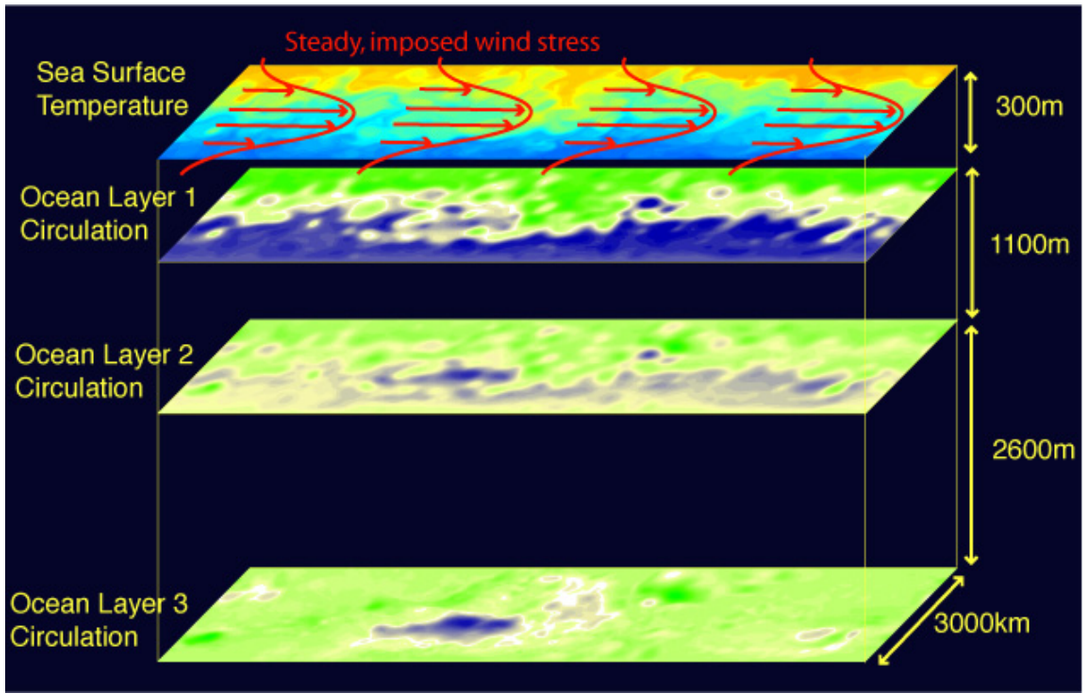
Summary

- **A consistent stochastic model :**
  - **Physical conservation laws satisfied**
  - **Better ensemble spread represented**
  - **Time-averaged profil well described on coarse mesh**

- **Future works :**
  - **Multi-layers QG model under LU**

    (Q-GCM Projects : Hogg et al. 2003)



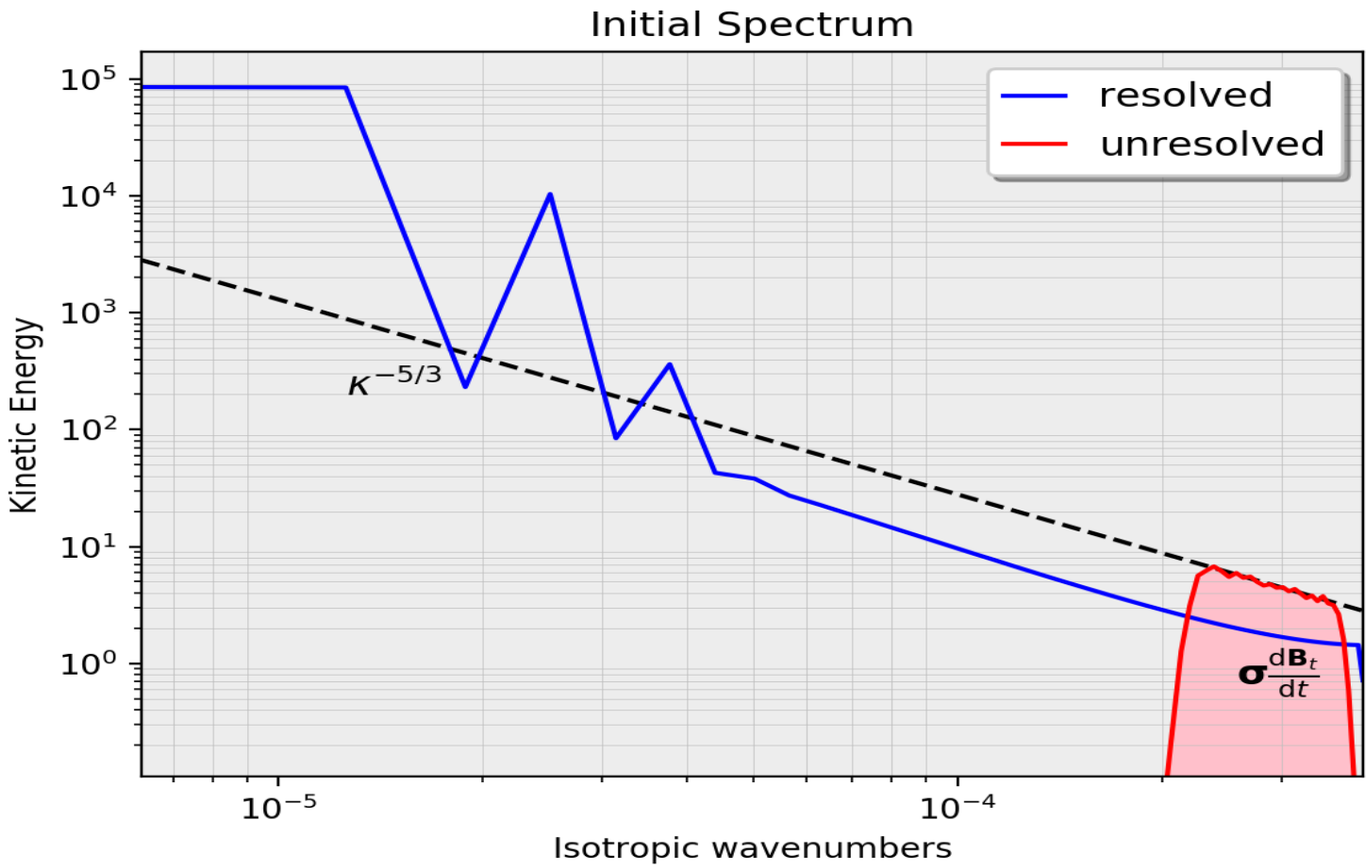


  - **Data Assimilation with Particle Filters**

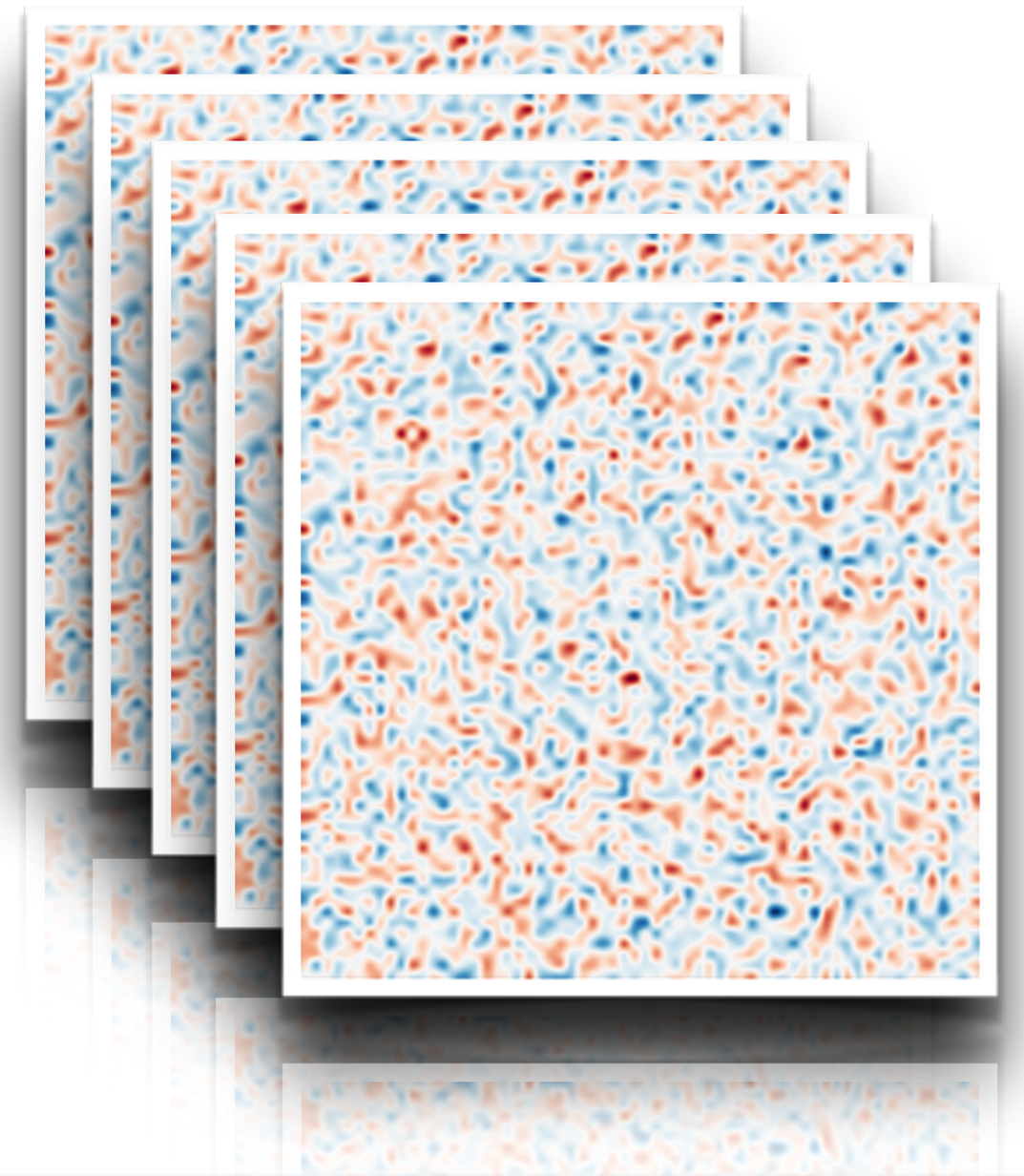
    (Cotter et al. 2018)

Thanks for Your Attention !

Homogeneous parameterization

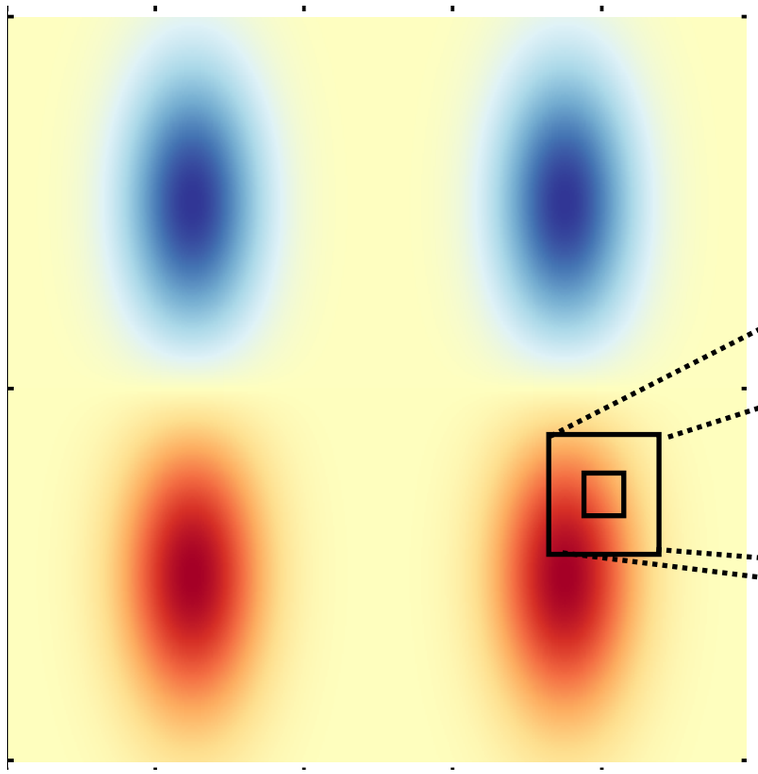


Ensemble of random stream functions



low-pass band filter $\star$ white noise

Heterogeneous parameterization

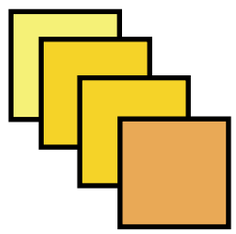


Neighbor

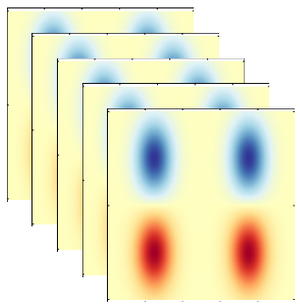
Centered neighbor

Random selection

Local ensemble :



Global ensemble



SVD

Noise